Constraint-Aware Design for Closed-Loop ILC Systems with Actuator Constraints

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Outline

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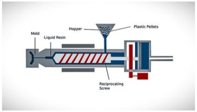
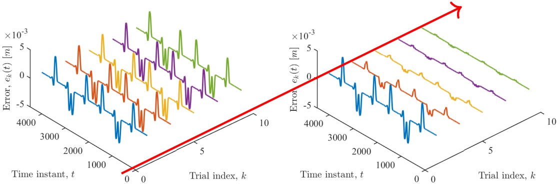
Iterative learning control (ILC)

- **Application examples**
  - **Gantry crane**
  - **Medical rehabilitation**
  - **Injection molding**
  - **Robotic arm**
- **Goal**
  - **Perfect tracking by ILC**
- **Insights**
  - **Repetitive**
  - **Learning**
- **Reduce repetitive disturbances!**

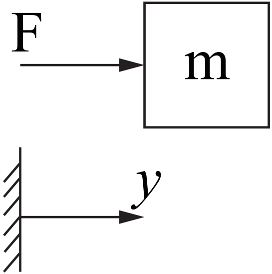




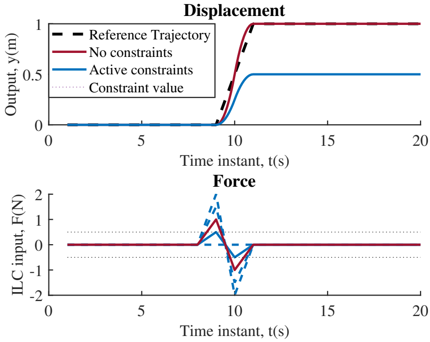
Feedback only **ILC**

Outline

- **Why constraint-aware ILC?**
  - **Mass example**
  - **Issues**
    - Integral windup in iteration domain
    - Lower learning efficiency

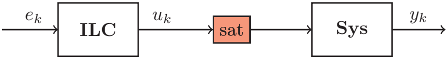


Mass example

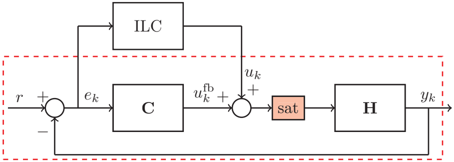


Input and output

- **Why constraint-aware ILC?**
  - **Mass example**
  - **Issues**
    - Integral windup in iteration domain
    - Lower learning efficiency
  - **Solution:** Enable ILC with constraint awareness
- **Input constraints:** open-loop ILC[1,2,⋯] vs. closed-loop ILC[3,4,⋯]



Open-loop ILC design



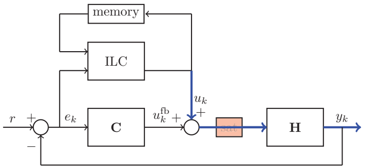
Closed-loop ILC design

[1] Ronghu Chi et al. "Constrained data-driven optimal iterative learning control". In: *J. Process Control* (2017).
[2] Matthew C Turner et al. "Anti-windup compensation for a class of iterative learning ⋯". In: *2023 ACC*. IEEE. 2023.
[3] Sandipan Mishra et al. "Optimization-based constrained iterative ⋯". In: *IEEE Trans. Control Syst. Technol.* (2010).
[4] Gijo Sebastian et al. "Convergence analysis of feedback-based iterative learning control ⋯". In: *Automatica.* (2019).

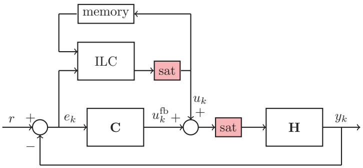
- **Process sensitivity** $u_k \to y_k$ (without constraints):

$$y_k = Gu_k, \tag{1}$$

- **Saturation constraint** $\Omega = \{u | -\bar{u} \le u(t) \le \bar{u},\ t \in [0, N-1]\}$
- **Issues when enabled with constraint awareness:**
  - **How the constraint on ILC Ω^{ff} affect the learning efficiency?**
  - **How to choose Ω^{ff}?**



Closed-loop ILC with actuator constraints



Constraint-aware ILC

Problem formulation

- **Process sensitivity** $u_k \to y_k$ (without constraints):

$$y_k = Gu_k, \tag{1}$$

- **Saturation constraint** $\Omega = \{u | -\bar{u} \le u(t) \le \bar{u},\ t \in [0, N-1]\}$
- **Issues when enabled with constraint awareness:**
  - **How the constraint on ILC Ω^{ff} affect the learning efficiency?**
  - **How to choose Ω^{ff}?**

Definition 1

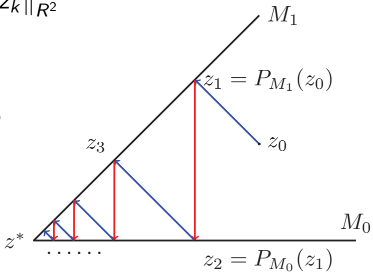
The **ILC design problem** is to find an ILC input sequence $\{u_{k+1}\}_{k\ge 0}$ under Ω^{ff} to solve the constrained optimization problem

$$\begin{aligned} &\min_{u_{k+1}\in\Omega^{\text{ff}}} \; J_{k+1}(u_{k+1}) \\ &\text{s.t. } \; e_{k+1} = r - Gu_{k+1}, \end{aligned} \tag{2}$$

such that e_{k+1} converges as k increases.

Constraint-aware ILC via alternating projections

- **Alternating projection problem** $\leftarrow$ **ILC design problem**
- **Example**
  - $H = R^2$
  - $z = (x, y)$ powered by **Cartesian product**
  - Two convex closed sets
    - $M_1 = \{(x, y) \in R^2 : y = x\}$
    - $M_0 = \{(x, y) \in R^2 : y = 0\}$
  - $z_{k+1} = P_{M_0/M_1}(z_k) \triangleq \arg \min_{z \in M_0/M_1} \|z - z_k\|^2_{R^2}$
  - $\{z_k\}_{k \geq 0}$ converges to $z^* = M_1 \cap M_0$
- **Extensions**
  - **High dimensions:** $x \in R^n$ and $y \in R^m$



Alternating projections in R^2

Constraint-aware ILC via alternating projections

- **Alternating projection problem $\leftarrow$ ILC design problem**
  - **Find two points minimizing the distance between**

$$M_1 = \{(e,u) \in H : e = r - y, y = Gu\}, \quad (3)$$

$$M_0 = \left\{(e,u) \in H : e = 0, u \in \Omega^{\mathrm{ff}}\right\}, \quad (4)$$

  - **Which set we put $u \in \Omega^{\mathrm{ff}}$?**[5]
    - M_1: **complex constrained optimization problem** $\min_{u \in \Omega^{\mathrm{ff}}} J_{k+1}$
    - M_0: **unconstrained optimization problem** $\min_{\hat{u}} J_{k+1}$, **and** $u = P_{\Omega^{\mathrm{ff}}}(u)$

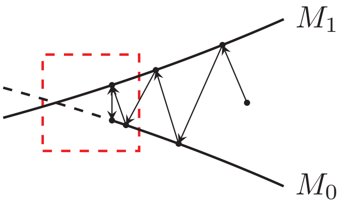


Illustration of alternating projections with input constraints

[5] Bing Chu et al. "Iterative learning control for constrained linear systems". In: *International Journal of Control* (2010).

- **Alternating projection problem** $\leftarrow$ **ILC design problem**
  - **Find two points minimizing the distance between**

$$M_1 = \{(e, u) \in H : e = r - y, y = Gu\}, \quad (3)$$
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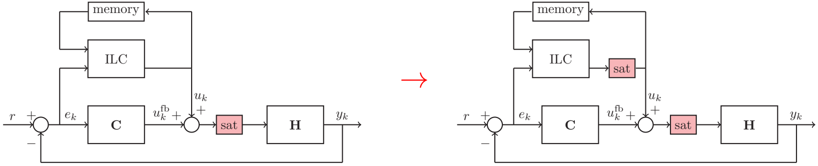
  - **Which set we put $u \in \Omega^{\mathrm{ff}}$?**

- **Chanlleges**
  - **How to analyze the learning efficiency?**
  - **How to choose Ω^{ff} subject to Ω?**



Traditional ILC under constraints **Constraint-aware ILC**

- **Constraint-aware ILC design**
  - **Projection implementation → Minimizing the cost function**

$$\min \left\| z_{\bar{k}+1} - z_{\bar{k}} \right\|_{H_C}^2 = \min_{u_{k+1} \in \Omega_{\mathrm{ff}}} J_{k+1}\left(u_{k+1}\right). \quad (5)$$

  - **Define the Hilbert space H_C:**

$$(e, u) \in H_C = \ell_2^m [1,\ N] \times \ell_2^l [0,\ N-1], \quad (6)$$

$$\langle (e, u), (e, v) \rangle_{\{Q,R\}} = e^T Q z + u^T R v, \quad (7)$$

$$\|(e, u)\|_{\{Q,R\}} = \sqrt{\langle (e, u), (e, u) \rangle_{\{Q,R\}}},\ Q \succ 0,\ R \succeq 0. \quad (8)$$

  - **Constraint-aware ILC update law**

$$u_{k+1} = P_{\Omega^{\mathrm{ff}}}\left(f\left(P_{\Omega^{\mathrm{ff}}}\left(u_k\right), e_k\right)\right), \quad (9)$$

where $P_{\Omega^{\mathrm{ff}}}(\cdot)$ is the projection operator and $f(\cdot)$ is the solution of (5).

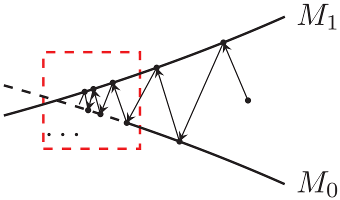
Constraint-aware ILC design

- **Learning efficiency analysis**

Theorem 3.1

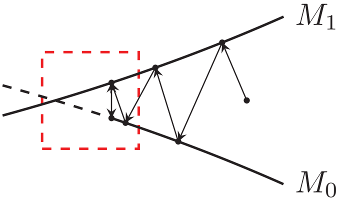
Given active Ω^{ff}, applying the constraint-aware ILC (9) yields the tracking error e_k converging with at most $\mathcal{K}+1$ trials, where for any initial point $z_0 = (e_0, u_0)$ in H_C and some $\alpha \in (0,1)$,

$$\mathcal{K} = \left\lfloor \log_{1-\alpha^2}\left(\frac{\text{dis}(M_1, M_0)}{\text{dis}(z_0, M_0)}\right)\right\rfloor. \quad (10)$$



Traditional ILC under constraints

→



Constraint-aware ILC

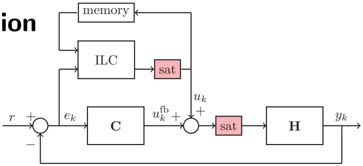
Constraint-aware ILC design

- **Choice of the constraint-aware value**

$$\begin{aligned} \min_{u_{k+1},\bar{u}^{\mathrm{c}}_{k+1}} \quad & J_{k+1}\left(u_{k+1},\bar{u}^{\mathrm{c}}_{k+1}\right) \\ \text{s.t.} \quad & e_{k+1} = \mathcal{S}r - GP_{\Omega_{k+1}}(u_{k+1}), \end{aligned} \tag{11}$$

where $\Omega^{\mathrm{ff}}_{k+1} = \left\{u \middle| -\bar{u}^{\mathrm{c}}_{k+1} \leq u(t) \leq \bar{u}^{\mathrm{c}}_{k+1},\ \forall t\right\}$ and $\mathcal{S}: r \to e_k$.

- **Two-step iterative optimization method**
  - **Given initial u_0 and $\bar{u}^{\mathrm{c}}_0$**
  - **Update u_{k+1} by the unconstrained ILC law $f(\cdot)$**
  - **Project u_{k+1} onto the known Ω^{ff}_k**
  - **Solve the single-variable optimization problem from (11) to get $\bar{u}^{\mathrm{c}}_{k+1}$**
  - **Project u_{k+1} onto $\Omega^{\mathrm{ff}}_{k+1}$**

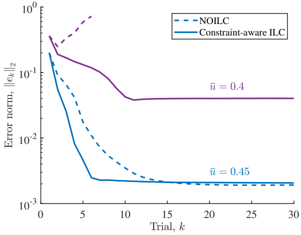


Constraint-aware ILC

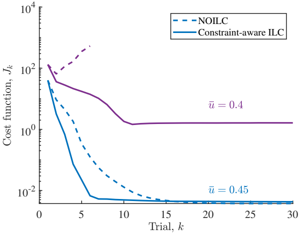
- **Transfer function of the plant model**

$$\mathrm{H}(s) = \frac{0.12s + 235}{0.00009s^4 + 0.01092s^3 + 21.385s^2} \quad (12)$$

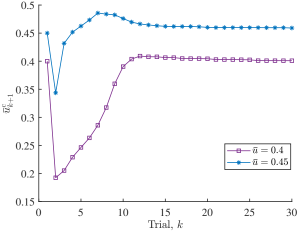
- **Stabilizing feedback controller C**
- **Unconstrained ILC law: NOILC**



Error



Cost



Constraint-aware value

Outline

Conclusion and Future Work

- **Advantages**
  - **Restrictions on the learning of ILC against instability**
  - **Constraint-aware design for improved learning efficiency**
- **Insights**
  - **Closed-loop ILC**
  - **Handling ILC input constraints in practice**
  - **Linear design for non-linear dynamics (constraint non-linearity)**
- **Application scenarios**
  - **Piezo-stepper actuator for nano-manufacturing**
  - **Upper limb rehabilitation**
  - ······

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