Alternating Projection-Based Iterative Learning Control for Repetitive Systems with Varying Trial Lengths and Practical Input Constraints

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Outline

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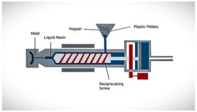
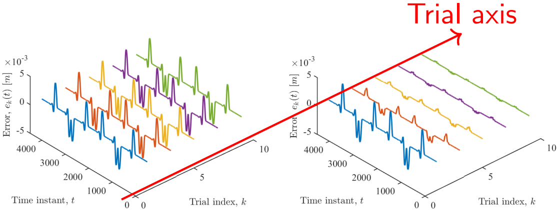
Iterative learning control (ILC)

- **Application examples**
  - **Gantry crane**
  - **Medical rehabilitation**
  - **Injection molding**
  - **Robotic arm**
- **Goal**
  - **Perfect tracking by ILC**
- **Insights**
  - **Repetitive**
  - **Learning**
- **Reduce repetitive disturbances!**



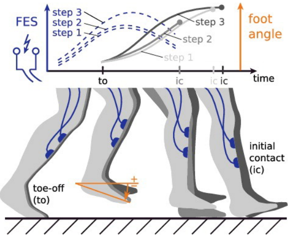


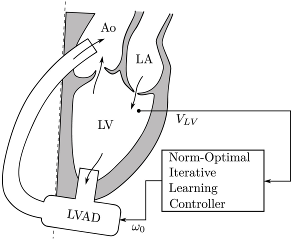
Feedback only **ILC**

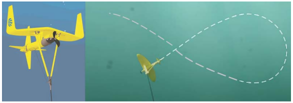
Outline

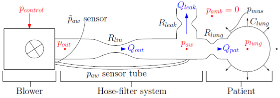
Repetitive systems with varying trial lengths

- **Foot motion assist device**[1]
- **Left ventricular assist device**[2]
- **Marine hydrokinetic energy system**[3]
- **Mechanical ventilator**[4]









[1] Thomas Seel et al. "Monotonic convergence of iterative learning control systems · · ·". In: *Int. J. Control.* (2017).
[2] Maike Ketelhut et al. "Iterative learning control of ventricular assist devices · · ·". In: *Control Eng. Pract.* (2019).
[3] Mitchell Cobb et al. "Flexible-time receding horizon iterative learning · · ·". In: *IEEE Trans. Control Syst. Technol.* (2022).
[4] Joey Reinders et al. "Triggered repetitive control: Application to · · ·". In: *IEEE Trans. Control Syst. Technol.* (2023).

Varying trial length problem

- **Missing information for learning**
  - **Extra design for learning efficiency**

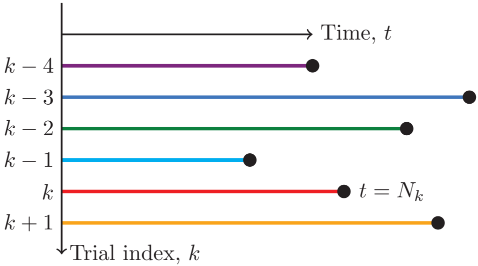
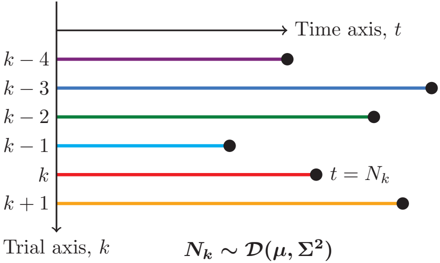


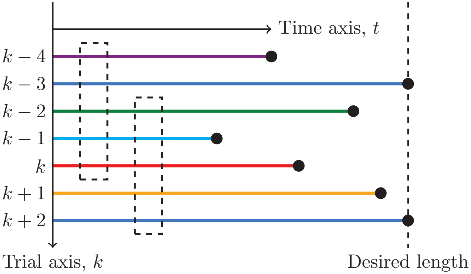
Illustration of varying trial length problem

Varying trial length problem

- **Missing information for learning**
  - **Extra design for learning efficiency**
    - **Model assumption** (Stochastic[5,6,⋯], deterministic[7,8,⋯])



Stochastic model



Deterministic model

[5] Xuefang Li et al. "An iterative learning control approach for linear systems ⋯". In: *IEEE Trans. Autom. Control.* (2014)
[6] Dong Shen et al. "On almost sure and mean square convergence of P-type ILC ⋯". In: *Automatica.* (2016)
[7] Thomas Seel et al. "Monotonic convergence of iterative learning control systems ⋯". In: *Int. J. Control.* (2017)
[8] Deyuan Meng et al. "Deterministic convergence for learning ⋯". In: *IEEE Trans. Neural Netw. Learn. Syst.* (2018)

Varying trial length problem

- **Missing information for learning**
  - **Extra design for learning efficiency**
    - **Model assumption** (Stochastic, deterministic)
    - **Information compensation** (zero[5], prediction[6,7], no compensation[8,9], $\cdots$)

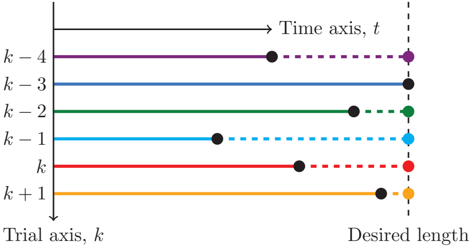
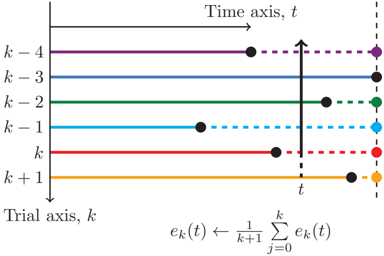


Illustration of compensations

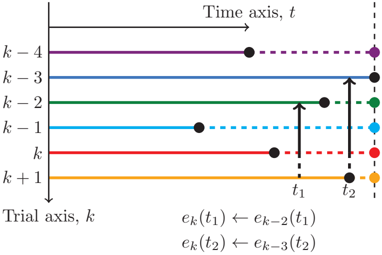
[5] Dong Shen et al. "On almost sure and mean square convergence of P-type ILC $\cdots$". In: *Automatica.* (2016)
[6] Na Lin et al. "Auxiliary predictive compensation-based ILC $\cdots$". In: *IEEE Trans. Syst., Man, Cybern., Syst.* (2019).
[7] Lele Ma et al. "Event-based switching iterative learning model predictive control $\cdots$". In: *IEEE Trans. Cybern.* (2023).
[8] Xu Jin. "Iterative learning control for MIMO nonlinear systems with $\cdots$". In: *IEEE Trans. Cybern.* (2021).
[9] Chen Liu et al. "Optimal learning control scheme for discrete-time $\cdots$". In: *IEEE Transactions on Cybernetics* (2022).

Varying trial length problem

- **Missing information for learning**
  - **Extra design for learning efficiency**
    - **Model assumption** (Stochastic, deterministic)
    - **Information compensation** (zero, prediction, no compensation, $\cdots$)
    - **Design mechanisms** (iteration-averaging[5], most recent one-order[6], event-based switching[7], optimal design[8,9,⋯], $\cdots$)



Iteration-averaging



Most recent one-order

[5] Xuefang Li et al. "An iterative learning control approach for linear systems $\cdots$". In: *IEEE Trans. Autom. Control.* (2014).
[6] Xu Jin. "Iterative learning control for MIMO nonlinear systems with $\cdots$". In: *IEEE Trans. Cybern.* (2021).
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Varying trial length problem

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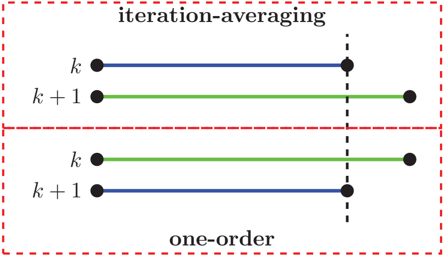

Event-based switching

[5] Xuefang Li et al. "An iterative learning control approach for linear systems $\cdots$". In: *IEEE Trans. Autom. Control.* (2014).
[6] Xu Jin. "Iterative learning control for MIMO nonlinear systems with $\cdots$". In: *IEEE Trans. Cybern.* (2021).
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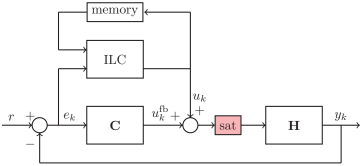
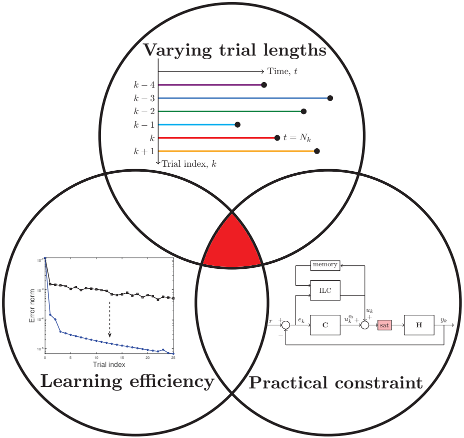
Varying trial length problem

- **Missing information for learning → Optimization-based ILC**
  - **Extra design for learning efficiency**
    - **Model assumption** (Stochastic, deterministic)
    - **Information compensation** (zero, prediction, no compensation, $\cdots$)
    - **Design mechanisms** (iteration-averaging, most recent one-order, event-based switching, optimal design, $\cdots$)
  - **Modified convergence analysis**
    - **Contraction mapping** (linear or globally Lipschitz continuous non-linear systems)
    - **Lyapunov-based composite energy function** (locally Lipschitz continuous non-linear systems)
    - **Variational analysis** (fractional order systems)
- **Practical input constraints → Constraint-aware ILC**



Why alternating projection-based design?

- **Alternating projection-based design**
  - **Intuitively and customizably geometric interpretation of problem**
  - **Hilbert space-enabled optimization methods**
  - **Practical constraint handling**



Outline

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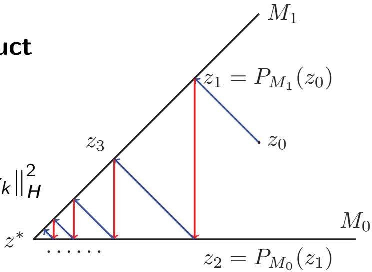
#0 Alternating projections in Hilbert space

- **Example**
  - $H = R^2$
  - $z = (x, y)$ powered by **Cartesian product**
  - Two convex sets
    - $M_1 = \{(x, y) \in R^2 : y = x\}$
    - $M_0 = \{(x, y) \in R^2 : y = 0\}$
  - $z_{k+1} = P_{M_0,M_1}(z_k) \triangleq \arg \min_{z \in M_0, M_1} \|z - z_k\|_H^2$
  - $\{z_k\}_{k \geq 0}$ converges to $z^* = M_1 \cap M_0$
- **Extensions**
  - **High dimensions:** $x \in R^n$ and $y \in R^m$
  - **More sets:** $M_0, M_1, M_2, \cdots, M_J$
- **vs. ILC**
  - **Proximity algorithm:** iterate to find a solution (**Learning**)
  - **Full model inverse for one step convergence:** $z^* = P_{M_1 \cap M_2}(z_0)$
  - **Projection:** optimal ILC design



Alternating projections in R^2

Outline

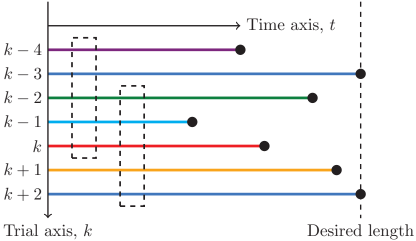
#1 Problem formulation

- **Motivation:** Optimal ILC design for learning efficiency
- **Lifted system with varying trial lengths**

$$\begin{cases} y_k = Gu_k,\, u_k \in \ell_2^l[0,\ N-1],\ y_k \in \ell_2^m[1,\ N], \\ e_k = F_k\,(r - y_k), \\ F_k = \begin{bmatrix} I_{N_k} \otimes I_m & 0 \\ 0 & 0_{N_d - N_k} \otimes 0_m \end{bmatrix}. \end{cases} \tag{1}$$

$$e_k = \Bigg[\overbrace{\underbrace{e_k^{\mathrm{T}}(1), \cdots, e_k^{\mathrm{T}}(N_k),}_{N_k} 0, \cdots, 0}^{N_d}\Bigg]^{\mathrm{T}}$$

Zero compensation



Deterministic model assumption

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Definition 1.1

The **ILC design problem** is to find an ILC update law

$$u_{k+1} = f(e_k, e_{k-1}, \cdots, u_k, u_{k-1}, \cdots), \quad (2)$$

for zero convergence of the modified tracking error in (1), i.e.,

$$\lim_{k \to \infty} \|e_k\| = 0. \quad (3)$$

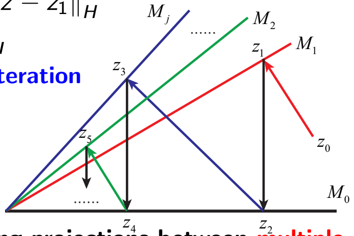
#1 Alternating projection-based ILC using multiple sets

- **Alternating projection problem** $\leftarrow$ **ILC design problem**
  - **design a projection order** to find a point in the intersection of:

$$\begin{aligned} M_j &= \{(e,u) \in H : e = F_j(r-y), y = Gu\} \in \{M_1, \cdots, M_J\}, \\ M_0 &= \{(e,u) \in H : e = 0\}. \end{aligned} \tag{4}$$

    - M_j **system dynamics**
    - M_0 **tracking objective**
  - **Projection operator:** $P_j(z) \triangleq \arg\min_{\hat{z} \in M_j} \|\hat{z} - z\|_H^2$

    minimize the "distance" between a point and a set in H
  - **Example:** $z_2 = P_0(z_1) \triangleq \arg\min_{\hat{z} \in M_0} \|\hat{z} - z_1\|_H^2$
  - **Projection on** M_0: no change on u
    - **Projecting on** M_j $\rightarrow$ **One ILC iteration**
    - **Constraint handling: See #3**



Alternating projections between multiple sets

#1 Alternating projection-based ILC using multiple sets

- **Alternating projection problem** $\leftarrow$ **ILC design problem**
  - **design a projection order** to find a point in the intersection of:

$$\begin{aligned} M_j &= \{(e,u) \in H : e = F_j(r-y), y = Gu\} \in \{M_1, \cdots, M_J\}, \\ M_0 &= \{(e,u) \in H : e = 0\}. \end{aligned} \tag{4}$$

    - M_j **system dynamics**
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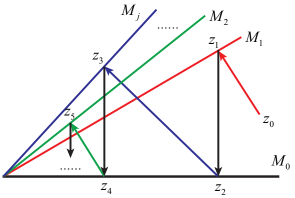


- **Challenges**
  - **How to design a projection order?**
  - **How to implement the projection?**
- **Notations**
  - **Index sequence**: $\{j_{\bar{k}}\}_{\bar{k} \geq 0}$ where $j_{\bar{k}} \in \{0, 1, 2, \cdots, J\}$.
  - **Projection sequence**: $\{z_{\bar{k}}\}_{\bar{k} \geq 0}$ by $z_{\bar{k}+1} = P_{j_{\bar{k}+1}}(z_{\bar{k}})$.

- **Projection order design**
  - **Necessary assumptions**
    - Closed convex sets
    - Infinitely many times

Definition 1.2

The sequence $s = \{j_{\bar{k}}\}_{\bar{k}\geq 0}$ taking i infinitely many times yields

$$\delta(s,i) = \sup_n \left[\Delta_{n+1}(i) - \Delta_n(i)\right] < \infty, \tag{5}$$

where $\{\Delta_n(i) \in \mathbb{N}\}_{n\geq 0}$ is an increasing sequence such that, at the n-times, $j_{\Delta_n(i)} = i$ with $\Delta_0(i) = 0$.

$\bar{k}$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	$\cdots$	$\delta(s,1)$	$\delta(s,2)$	$\delta(s,3)$
$j_{\bar{k}}$	3	1	1	2	3	1	3	2	$\cdots$	3	4	4

Table. Example with $J = 3$ until $k = 8$.

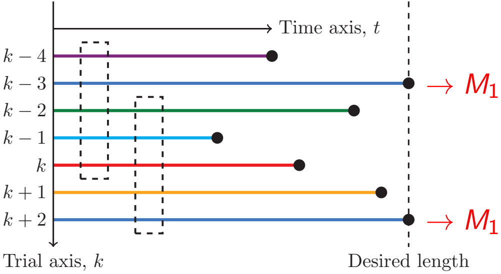
#1 Projection order design

Assumption 1.1

Let F_1 has full row rank and $M_J \subseteq \cdots \subseteq M_2 \subseteq M_1$. M_1 appears infinitely many times during the alternating projections between M_j and M_0, i.e.

$$\delta(s,1) = \sup_n \left[\Delta_{n+1}(1) - \Delta_n(1)\right] < \infty. \tag{6}$$

- Assumption 1.1 $\leftarrow$ Deterministic model assumption
- **Full learning property**

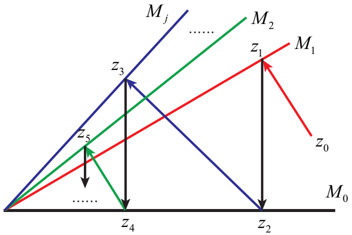


- **Projection order design**
  - **Necessary assumption**: Assumption 1.1 (full learning property)
  - **Projection order**

$$M_{j_{\bar{k}}} = \begin{cases} M_j \in \{M_1, M_2, \ldots, M_J\}, & \bar{k} \text{ is odd,} \\ M_0, & \bar{k} \text{ is even.} \end{cases} \tag{7}$$

  - **Projection sequence**

$$\{z_{\bar{k}}\}_{\bar{k}\geq 0} : \begin{cases} z_{2\bar{k}+1} = P_{j_{2\bar{k}+1}}(z_{2\bar{k}}), \\ z_{2\bar{k}} = P_{j_{2\bar{k}}}(z_{2\bar{k}+1}). \end{cases} \tag{8}$$



Projection order design

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  - **Necessary assumption**: Assumption 1.1 (full learning property)
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  - **Convergence analysis:** Alternating projections under (7).

Theorem 1.1

The sequence $\{z_{\bar{k}}\}_{\bar{k} \geq 0}$ converges in norm to the orthogonal projection of z_0 onto $M_j \cap M_0$ under the projection order (7).[a]

[a] Zhihe Zhuang et al. "Alternating projection-based iterative learning control for discrete-time systems with non-uniform trial lengths". In: *International Journal of Robust and Nonlinear Control* (2023).

#1 Optimal ILC algorithms

- **Optimal ILC algorithm** $\leftarrow$ **Projection implementation**
  - **Projection implementation $\rightarrow$ Minimizing the cost function**

$$\min \left\| P_{j_{\bar{k}+1}}(z_{\bar{k}}) - z_{\bar{k}} \right\|_H^2 \rightarrow \min \ J_{k+1}. \tag{9}$$

  - **Define H by inner product and associated induced norm**:

$$(e, u) \in H = \ell_2^m [1,\ N] \times \ell_2^l [0,\ N-1], \tag{10}$$

$$\langle (e, u), (y, v) \rangle_{\{Q,R\}} = \sum_{i=1}^{N_d} e^T(i)\, Q y(i) + \sum_{i=0}^{N_d-1} u^T(i) R v(i), \tag{11}$$

$$\|(e, u)\|_{\{Q,R\}} = \sqrt{\langle (e, u), (e, u) \rangle_{\{Q,R\}}},\ Q \succ 0,\ R \succeq 0. \tag{12}$$

  - **Optimal ILC update law** $\leftarrow J_{k+1} = \|e_{k+1}\|_Q^2 + \|u_{k+1} - u_k\|_R^2$

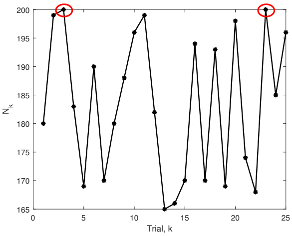
$$u_{k+1} = u_k + L e_k, \tag{13}$$

where $L = (G^{\mathrm{T}} Q G + R)^{-1} G^{\mathrm{T}} Q$.

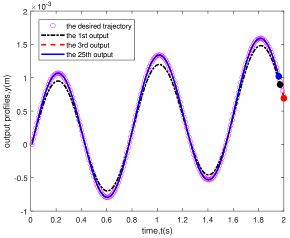
- **Transfer function**

$$G(s) = \frac{15.8869\,(s + 850.3)}{s\left(s^2 + 707.6s + 3.377 \times 10^5\right)}, \tag{14}$$

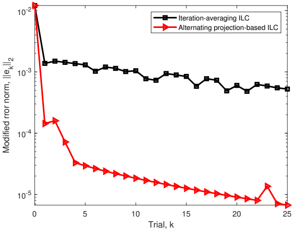
- Sampling time 0.01s, operation time 2s, desired length $N_d = 200$.
- Set $N_k \sim U(165, 200)$ where $\delta(s,1) = 20$, $N_3 = 200$, and $N_{23} = 200$.



N_k



Output



Error norm

#1 Summary

- **Advantages**
  - **Optimal design without learning gain tuning**
    - Weighting parameters Q and R vs. Arimoto-type learning gain
  - **Straightforward but effective mechanisms**
    - Zero compensation
    - Most recent one-order learning by lifted framework
  - **Convergence guarantee under alternating projections**
- **Insights**
  - **Special case:** NOILC for linear systems with varying trial lengths
  - **Allow more design freedom:** More numerial optimization methods
  - **Extensions to other non-repetitive ILC problems**
    - Trial-varying tracking references
    - Nonidentical initial state
    - Trial-varying system plant
    - $\cdots\cdots$

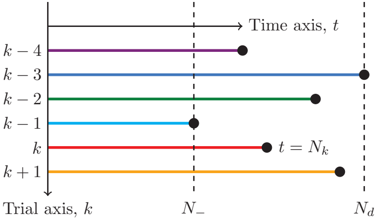
Outline

#2 Problem formulation

- **Motivation:** Optimal ILC design using probability information
- **Lifted system with varying trial lengths**

$$\begin{cases} y_k = Gu_k, u_k \in \ell_2^l[0,\ N-1],\ y_k \in \ell_2^m[1,\ N], \\ e_k = F_k\,(r - y_k), \\ F_k = \begin{bmatrix} I_{N_k} \otimes I_m & 0 \\ 0 & 0_{N_d - N_k} \otimes 0_m \end{bmatrix}. \end{cases} \tag{15}$$

- **Random variable $N_k \sim \mathcal{D}(N_-, N_d)$**



Stochastic model

- **Motivation:** Optimal ILC design using probability information
- **Lifted system with varying trial lengths**

$$\begin{cases} y_k = Gu_k, u_k \in \ell_2^l[0,\ N-1],\ y_k \in \ell_2^m[1,\ N], \\ e_k = F_k\left(r - y_k\right), \\ F_k = \begin{bmatrix} I_{N_k} \otimes I_m & 0 \\ 0 & 0_{N_d - N_k} \otimes 0_m \end{bmatrix}. \end{cases} \tag{15}$$

- **Random variable $N_k \sim \mathcal{D}(N_-, N_d)$**
  - $P(N_k = N_i) = p_i$ where $\sum_{i=1}^{N_d - N_- + 1} p_i = 1$.
  - **Stochastic information used:** Mathematical expectation of F_k

$$\begin{aligned} \bar{F} &\triangleq E\left\{F_k\right\} \\ &= \mathrm{diag}\left\{\overbrace{1, \cdots, 1}^{N_- - 1}, p\left(N_k = N_-\right), \cdots, p\left(N_k = N_d\right)\right\} \otimes I_m. \end{aligned} \tag{16}$$

#2 Stochastic-optimization ILC via alternating projections

Definition 2.1

The **ILC design problem** is to find an ILC update law

$$u_{k+1} = f(e_k, e_{k-1}, \cdots, u_k, u_{k-1}, \cdots), \tag{17}$$

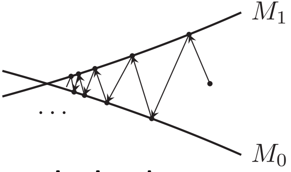
such that $\lim_{k\to\infty} \|E\{e_k\}\| = 0$.

- **Alternating projection problem** ← **ILC design problem**

$$M_1 = \{(\underline{e}, u) \in H_E : \underline{e} = E\{F(r-y)\}, y = Gu\}, \tag{18}$$

$$M_0 = \{(\underline{e}, u) \in H_E : \underline{e} = 0\}, \tag{19}$$

$$H_E = \ell_2^m[1,\ N] \times \ell_2^l[0,\ N-1] \tag{20}$$



Alternating projections between two sets

#2 Stochastic-optimization ILC algorithm

- **Stochastic-optimization ILC algorithm**
  - **Projection implementation → Minimizing the cost function**

$$\min \left\| z_{\bar{k}+1} - z_{\bar{k}} \right\|_{H_E}^2 = \min J_{k+1}^E \tag{21}$$

  - **Define the Hilbert space H_E:**

$$\langle (\underline{e}, u), (\underline{e}, v) \rangle_{\{Q,R\}} = \underline{e}^T Q \underline{z} + u^T R v, \tag{22}$$

$$\| (\underline{e}, u) \|_{\{Q,R\}} = \sqrt{\langle (\underline{e}, u), (\underline{e}, u) \rangle_{\{Q,R\}}}, \; Q \succ 0, \; R \succeq 0. \tag{23}$$

  - **Stochastic-optimization ILC** $\leftarrow J_{k+1}^E = \| E\{e_{k+1}\} \|_Q^2 + \| u_{k+1} - u_k \|_R^2$

Theorem 2.1

Minimizing J_{k+1}^E has a feedforward solution

$$u_{k+1} = u_k + L_E e_k, \tag{24}$$

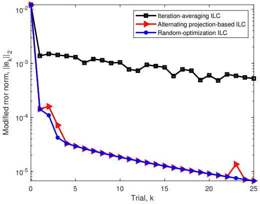
where $L_E = \left(G^{\mathrm{T}} K G + R \right)^{-1} G^{\mathrm{T}} \bar{F}^{\mathrm{T}} Q$ and $K = E\left\{ F_k^{\mathrm{T}} Q F_k \right\}$.[a]

[a] Zhihe Zhuang et al. "Iterative learning control for repetitive tasks with randomly varying trial lengths using successive projection". In: *Int. J. Adapt. Control Signal Process.* (2022).

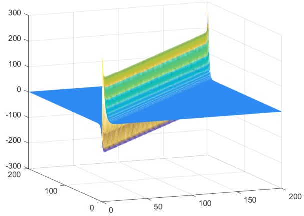
- **Transfer function**

$$G(s) = \frac{15.8869\,(s+850.3)}{s\left(s^2+707.6s+3.377\times 10^5\right)}, \tag{25}$$

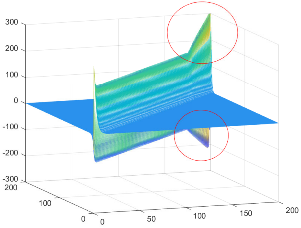
- Sampling time 0.01s, operation time 2s, desired length $N_d = 200$.
- Set $N_k \sim U(165, 200)$ where $\delta(s,1) = 20$, $N_3 = 200$, and $N_{23} = 200$.



Error norm



#1 L



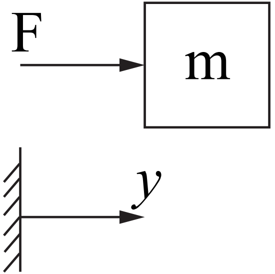
#2 L_E

#2 Summary

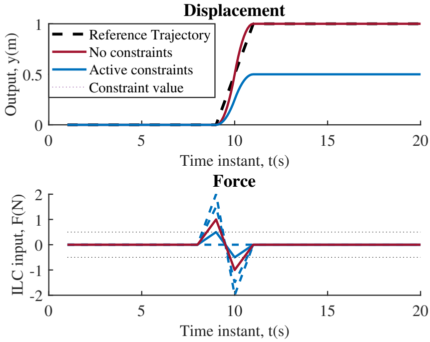
- **Advantages**
  - **Optimal design without learning gain tuning**
    - Weighting parameters Q and R vs. Arimoto-type learning gain
  - **Straightforward but effective mechanisms**
    - Zero compensation
    - Most recent one-order learning by lifted framework
  - **Convergence guarantee under alternating projections**
  - **Further optimization using probility information**
- **Insights**
  - **More information used for optimization**
  - **Modified weights in learning gain**
  - **Extensions to other stochastic factors**
    - Non-repetitive disturbances with known probility information
    - ......

Outline

- **Why constraint-aware ILC?**
  - **Mass example**
  - **Issues**
    - Integral windup in iteration domain
    - Lower learning efficiency

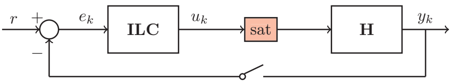


Mass example

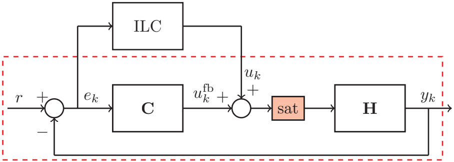


Input and output

- **Why constraint-aware ILC?**
  - **Mass example**
  - **Issues**
    - Integral windup in iteration domain
    - Lower learning efficiency
  - **Solution:** Enable ILC with constraint awareness
- **Input constraints:** Direct ILC[5,6,⋯] vs. Indirect ILC (separately)[7,8,⋯]



Direct ILC design



Indirect ILC design

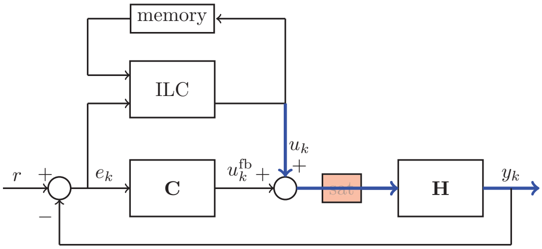
[5] Ronghu Chi et al. "Constrained data-driven optimal iterative learning control". In: *J. Process Control* (2017).
[6] Matthew C Turner et al. "Anti-windup compensation for a class of iterative learning ⋯". In: *2023 ACC*. IEEE. 2023.
[7] Sandipan Mishra et al. "Optimization-based constrained iterative ⋯". In: *IEEE Trans. Control Syst. Technol.* (2010).
[8] Gijo Sebastian et al. "Convergence analysis of feedback-based iterative learning control ⋯". In: *Automatica.* (2019).

- **Process sensitivity** $u_k \to y_k$ (without constraints):

$$y_k = Gu_k, \tag{26}$$

where the input constraint for ILC $u_k \in \Omega_{\text{ff}}$ is unknown subject to:

  - **Actuator constraints**
  - **Extra non-repetitive disturbances**: $u_k + u_k^{\text{fb}} \in \Omega$



Closed-loop control block diagram

#3 Problem formulation

- **Process sensitivity** $u_k \to y_k$ (without constraints):

$$y_k = Gu_k, \tag{26}$$

where the input constraint for ILC $u_k \in \Omega_{\mathrm{ff}}$ is unknown subject to:

- **Actuator constraints**
- **Extra non-repetitive disturbances**: $u_k + u_k^{\mathrm{fb}} \in \Omega$

Definition 3.1

The **ILC design problem** is to find a suitable Ω_{ff} to solve the constrained optimization problem

$$\begin{aligned} &\min_{u_{k+1} \in \Omega_{\mathrm{ff}}} \quad J_{k+1}\left(u_{k+1}\right) \\ &\text{s.t.} \;\; e_{k+1} = r - Gu_{k+1}, \end{aligned} \tag{27}$$

to find an ILC algorithm generating ILC input sequence $\{u_{k+1}\}_{k \geq 0}$ such that e_{k+1} converges as k increases.

#3 Constraint-aware ILC via alternating projections

- **Alternating projection problem** $\leftarrow$ **ILC design problem**
  - **Find two points minimizing the distance between**

$$M_1 = \{(e, u) \in H : e = r - y, y = Gu\}, \tag{28}$$

$$M_0 = \{(e, u) \in H : e = 0, u \in \Omega_{\mathrm{ff}}\}, \tag{29}$$

  - **Which set we put** $u \in \Omega_{\mathrm{ff}}$**?**[9]
    - M_1**: complex constrained optimization problem** $\min_{u \in \Omega_{\mathrm{ff}}} J_{k+1}$
    - M_0**: unconstrained optimization problem** $\min_{\hat{u}} J_{k+1}$**, and** $u = P_{\Omega_{\mathrm{ff}}}(u)$

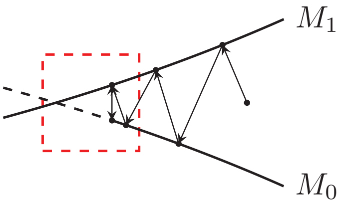


Illustration of alternating projections with input constraints

[9] Bing Chu et al. "Iterative learning control for constrained linear systems". In: *International Journal of Control* (2010).

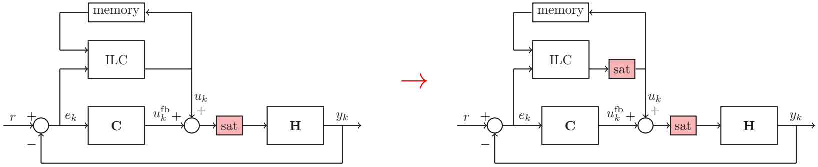
#3 Constraint-aware ILC via alternating projections

- **Alternating projection problem ← ILC design problem**
  - **Find two points minimizing the distance between**

$$M_1 = \{(e, u) \in H : e = r - y, y = Gu\}, \tag{28}$$
$$M_0 = \{(e, u) \in H : e = 0, u \in \Omega_{\text{ff}}\}, \tag{29}$$

  - **Which set we put $u \in \Omega_{\text{ff}}$?**

- **Chanlleges**
  - **How to settle Ω_{ff} with respect to Ω? (Soft constraints?)**
  - **How to analyze the learning efficiency?**



Traditional ILC under constraints **Constraint-aware ILC**

- **Constraint-aware ILC design**
  - **Projection implementation → Minimizing the cost function**

$$\min \left\| z_{\bar{k}+1} - z_{\bar{k}} \right\|_{H_C}^{2} = \min_{u_{k+1} \in \Omega_{\mathrm{ff}}} J_{k+1}\left(u_{k+1}\right). \tag{30}$$

  - **Define the Hilbert space H_C:**

$$(e, u) \in H_C = \ell_2^m \left[1,\ N\right] \times \ell_2^l \left[0,\ N-1\right], \tag{31}$$

$$\left\langle (e, u), (e, v) \right\rangle_{\{Q,R\}} = e^T Q z + u^T R v, \tag{32}$$

$$\left\| (e, u) \right\|_{\{Q,R\}} = \sqrt{\left\langle (e, u), (e, u) \right\rangle_{\{Q,R\}}},\ Q \succ 0,\ R \succeq 0. \tag{33}$$

  - **Constraint-aware ILC update law**

$$u_{k+1} = P_{\Omega_{\mathrm{ff}}}\left(f\left(P_{\Omega_{\mathrm{ff}}}\left(u_k\right), e_k\right)\right), \tag{34}$$

where $P_{\Omega_{\mathrm{ff}}}(\cdot)$ is the projection operator and $f(\cdot)$ is the solution of (30).

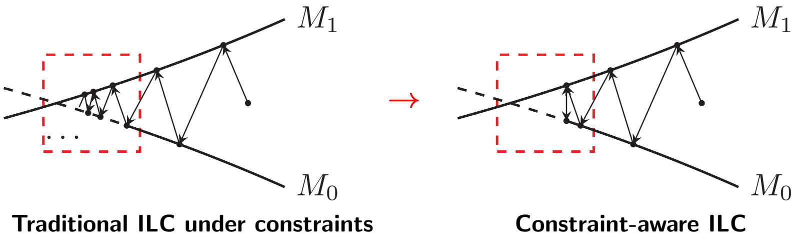
#3 Constraint-aware ILC analysis

- **Learning efficiency analysis**

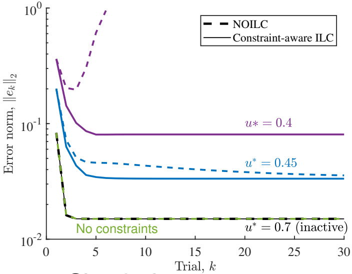
Theorem 3.1

Given the constraint set Ω, applying the constraint-aware ILC (34) yields the tracking error e_k converging with at most $\mathcal{K}+1$ trials under actuator saturation constraints, where for any initial point $z_0 = (e_0, u_0)$ in H_C and some $\alpha \in (0,1)$,

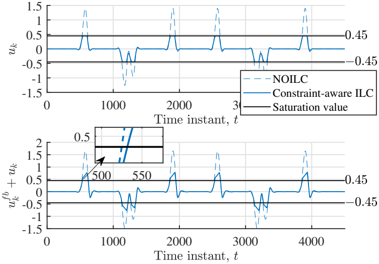
$$\mathcal{K} = \left\lfloor \log_{1-\alpha^2}\left(\frac{\mathrm{dis}(M_1, M_0)}{\mathrm{dis}(z_0, M_0)}\right)\right\rfloor. \tag{35}$$



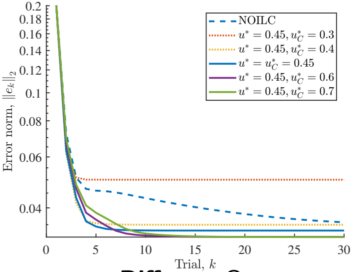
- **Simulation results**
  - **Stabilizing feedback controller**
  - **Compared to NOILC**
  - **Input profiles**
  - **Different choice of $\Omega_{\rm ff}$**



Simulation error norm

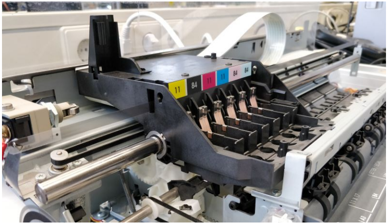


Input profiles

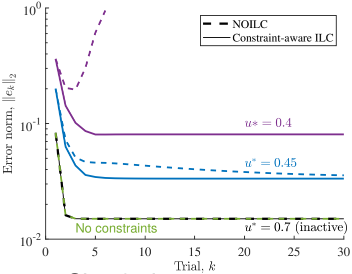


Different $\Omega_{\rm ff}$

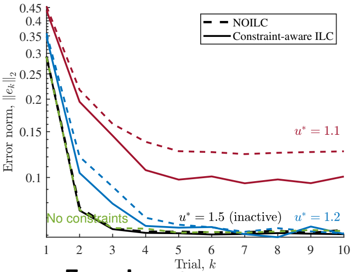
- **Simulation results**
  - Stabilizing feedback controller C
  - Compared to NOILC
  - Input profiles
  - Different choice of Ω_{ff}
- **Experimental results**
  - Desktop printer



Desktop printer



Simulation error norm



Experiment error norm

#3 Summary

- **Advantages**
  - **Restrictions on the learning of ILC against instability**
  - **Constraint-aware design for improved learning efficiency**
- **Insights**
  - **Indirect ILC architecture for constraint-aware design**
  - **Handling ILC input constraints in practice**
  - **Linear design for non-linear dynamics (constraint non-linearity)**
- **Application scenarios**
  - **Piezo-stepper actuator for nano-manufacturing**
  - **Upper limb rehabilitation**
  - ......

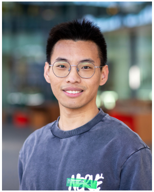
Outline

Conclusion and Future work

- **Conclusion**
  - **Optimal ILC for constrained systems with varying trial lengths**
  - **Constraint-aware ILC for practical input constraints**
  - **Improved learning efficiency via alternating projections**
- **Future work**
  - **Non-linear systems**
  - **Direct data-based perspective**
  - **Reinforcement learning-enabled design**
  - **Practical applications**

Outline

Acknowledgments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