

Iterative Learning Control for Closed-Loop Systems with Actuator Saturation using Alternating Projection

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Abstract: Iterative learning control (ILC) is typically applied in practice combined with a feedback controller for time-domain stability. In this closed-loop design with actuator constraints, existing constrained ILC designs suffer from determining the exact input constraint on the ILC controller. This issue brings in an important gap between the existing constrained ILC designs and their real-world applications. This paper gives a systematic consideration of the input constraint problem in the closed-loop ILC design with actuator saturation. A constraint-aware ILC is developed to autonomously determine the constraint on the feedforward controller. The convergence of the constrained ILC process is proved under the framework of alternating projection. Finally, the effectiveness of the developed method is verified on a numerical simulation.

Key Words: Iterative Learning Control, Input Constraint, Feedforward Control, Integral Windup

1 Introduction

Iterative learning control (ILC) aims at solving repetitive tasks within a limited time duration. The repetitive factors, including the reference signal and external unknown disturbances, can be handled by ILC. ILC has been applied to plenty of scenarios, such as batch process [1, 2], rehabilitation [3], and precision motion systems [4, 5]. Refer to [6–8] for the overview of ILC.

ILC is usually applied in the closed-loop design as a feedforward part, which combines a feedback controller to ensure stability in the time domain. Common closed-loop structures include parallel ILC and serial ILC [7, 9], both cases demand constraints on the actuator for safety or other reasons. Note that the design of ILC has a feedback mechanism in the trial domain. The ILC signal would accumulate trial by trial and bring in performance degradation in the closed-loop designs. Therefore, an appropriate restriction should be given on the ILC controller, which enables ILC with constraint awareness.

There are plenty of constrained ILC designs in the literature. In [10, 11], the input constraint is considered in the design of ILC update law, which can ensure the trial-domain error convergence. The hyperbolic tangent function is employed in [12, 13] to smoothly tackle input constraints so that it can improve the ILC performance under input constraints. Besides, an anti-windup compensator from the time domain is considered in the ILC systems to avoid instability in the

trial domain [14].

In addition, the ILC design with input constraints can be reformulated into a constrained optimization problem, which can be solved by numerical optimization methods [15, 16]. This optimization-based ILC (OBILC) design can address the input constraints in an effective manner. In [17, 18], the barrier method is incorporated in the ILC design to solve the constrained ILC problem under input constraints. The comparison of convergence speed between the gradient descent method and the Newton-based design is investigated in [19], where OBILC is considered to solve a point-to-point tracking problem under mixed constraints. In [21] and [20], the conic input mapping and the forward-backward splitting algorithm are respectively used in the ILC designs, where better control performance and faster convergence rate are obtained. However, the aforementioned methods are mainly considered in an open-loop perspective with given input constraints. When ILC is applied in a closed-loop structure together with a feedback controller, the actuator constraint is typically known rather than the constraint on the ILC controller directly.

In this paper, a constraint-aware ILC design is developed in the closed-loop parallel structure with actuator saturation using alternating projections. The trial-domain integral windup is avoided by enabling ILC with constraint awareness. The convergence performance is improved compared to the traditional norm optimal ILC (NOILC) without constraint-aware design. Also, a systematical consideration of input constraints is investigated in the closed-loop ILC design, where the constraint on the ILC controller is autonomously determined according to the given actuator saturation. The convergence analysis is given under the frame-

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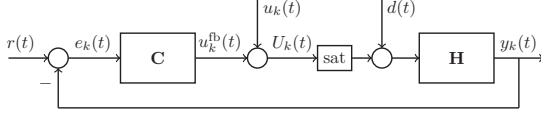


Fig. 1: The closed-loop control block diagram.

work of alternating projection, providing a holistic perspective on the constrained ILC design by geometric theory. The developed method is verified on a numerical example.

This paper is organized as follows. Section 2 gives the problem formulation to introduce the ILC problem considered in this paper. The constraint-aware ILC is developed in Section 3, where an OBILC design is given in the closed-loop design with actuator saturation. In Section 4, the developed method is verified by a numerical simulation. Finally, the conclusion and future work is given in Section 5.

The notations of the paper are given as follows. \mathbb{N} denotes the set of natural numbers. \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote the sets of n -D real vectors and $n \times m$ real matrices, respectively. The zero vector with appropriate dimensions is represented by the notation $\mathbf{0}$. $l_2[a, b]$ denotes the space of \mathbb{R}^1 valued Lebesgue square-summable sequences defined on an interval $[a, b]$. The superscript \top denotes the transpose. The notation $\langle \cdot \rangle$ denotes the inner product, and $\mathbb{X} \times \mathbb{Y}$ denotes the Cartesian product of two spaces \mathbb{X} and \mathbb{Y} . The notation $\text{sgn}(\cdot)$ denotes the signum/sign function. Other notations will be introduced once it occurs in the paper.

2 Problem Formulation

2.1 Preliminaries

Consider a closed-loop feedback control block diagram as shown in Fig. 1 that performs a repetitive task of finite time length $N < \infty$, where $t = 1, 2, \dots, N$ is a discrete-time index. The subscript $k \in \mathbb{N}$ denotes the trial index. In Fig. 1, a stabilizing feedback control $C(q^{-1})$ is first designed for the given linear time-invariant (LTI) system $\mathcal{H}(q^{-1})$ where $\mathcal{H}(q^{-1})$ and $C(q^{-1})$ are transfer functions represented by the time-forward operator q . The notations $u_k^{fb}(t)$, $u_k(t)$ and $y_k(t)$ are the k th outputs at time instant t of feedback controller, feedforward controller and the system, respectively. The total input from both feedback and feedforward controller is denoted by $U(t) = u_k^{fb}(t) + u_k(t)$, which is the input of the actuator. In practice, there usually exists a constraint block for the input constraint of the actuator. This paper considers a actuator saturation constraint, which is defined as

$$\text{sat}(U(t), \bar{u}) \triangleq \text{sgn}(U(t)) \min\{|U(t)|, \bar{u}\}, \quad (1)$$

where the actuator saturation is symmetrically constrained by an identical absolute limit \bar{u} for both upper and lower bounds. Since the actuator saturation is not always reached in practical applications, this paper focuses on the case of the active constraint, which is defined in the following definition.

Definition 1 (Active constraint) *The actuator saturation constraint is active if and only if there exists a $t \in [0, N-1]$ such that $\text{sat}(U(t), \bar{u}) = \bar{u}$.*

The goal of this paper is to develop an ILC algorithm to generate the feedforward signal $u_k(t)$ to reduce the effects

raised by the repetitive reference $r(t)$ and repetitive external disturbance $d(t)$.

2.2 System representation

This paper considers using the model information for the ILC designs. The *process sensitivity* is the mapping from the feedforward input $u_k(t)$ to the output $y_k(t)$ without constraints in Fig. 1. This mapping can be represented by the following state-space formulation:

$$\begin{cases} x_k(t+1) = Ax_k(t) + Bu_k(t), \\ y_k(t) = Cx_k(t), \end{cases} \quad (2)$$

where the state is denoted by $x_k(t) \in \mathbb{R}^n$, and the system parameter matrices are respectively denoted by A , B , and C . In this paper, the initial state is assumed to be identical, i.e., $x_k(0) = x_0$. The state-space formulation of system (2) can be transferred into the following lifted form:

$$y_k = Gu_k + d_k, \quad (3)$$

where

$$G = \begin{bmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ CA^2B & CAB & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{N-1}B & CA^{N-2}B & CA^{N-3}B & \cdots & CB \end{bmatrix},$$

and

$$d_k = \begin{bmatrix} (CA)^\top & (CA^2)^\top & \cdots & (CA^N)^\top \end{bmatrix}^\top x_k(0).$$

The lifted system matrix G is a N -length impulse response matrix of the process sensitivity. If the relative degree of the closed-loop system is $l = 1$, then $CB \neq 0$ holds for the considered single-input single-output (SISO) system and hence G is full-row-rank. For brevity, the discussion on $l > 1$ can be referred to in [15] and is omitted in this paper. All the repetitive factors, including the repetitive external disturbance $d(t)$ in Fig. 1 and the effect of the identical initial state d_k , can be considered as the part of components with respect to the repetitive reference. For brevity, the remainder of this paper will only consider the reference $r(t)$.

When the relative degree $l = 1$, the input and output vectors are respectively represented as

$$\begin{aligned} u_k &= [u_k(0), u_k(1), \dots, u_k(N-1)]^\top, \\ y_k &= [y_k(1), y_k(2), \dots, y_k(N)]^\top. \end{aligned}$$

The repetitive reference vector is defined as $r = [r(1), r(2), \dots, r(N)]^\top$, and then the tracking error vector is defined as

$$e_k = r - y_k. \quad (4)$$

In the presence of active actuator constraints, the ILC problem should be carefully solved, which will be elaborated in the next subsection.

2.3 ILC design problem

In the closed-loop design, ILC is open-loop in the time domain and typically no restriction is given for its trial-domain

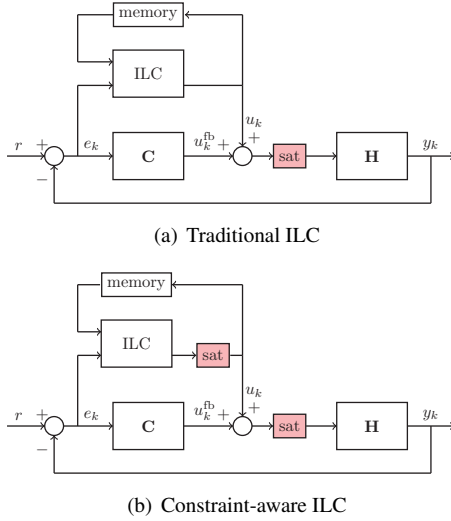


Fig. 2: Constraint-aware ILC in the closed-loop design.

feedback dynamics. Therefore, the integral windup in the trial domain could happen in the presence of active actuator constraints. In this case, the ILC controller tries to increase the input signals to compensate the error regardless of the saturation when the desired input is unattainable.

In the closed-loop design Fig. 1, the unlimited feedforward input $u(t)$ will also affect the feedback dynamics in the time domain and result in the time-domain integral windup. Therefore, the ILC signal should be appropriately constrained. The illustration of the developed constraint-aware ILC is given in Fig. 2, where the output of the feedforward controller is constrained by a constraint buffer. The ILC problem discussed in this paper mainly involves design this constraint buffer, which is concluded in the following definition.

Definition 2 *The ILC problem in this paper is to design an ILC algorithm to generate the ILC input together with its corresponding constraint \bar{u}^c in the closed-loop design with active actuator constraints such that the error can converge as $k \rightarrow \infty$.*

In the next section, the ILC design is given under the framework of alternating projections.

3 Constraint-Aware ILC Design

The trial-domain dynamics of the ILC process is investigated in the high-dimensional Hilbert space, which provides a holistic perspective on the optimization-based design via alternating projections. The input and output vectors are respectively defined in the Hilbert spaces $u_k \in l_2[0, N-1]$ and $y_k \in l_2[1, N]$, whose inner products and associated induced norms are given as follows:

$$\langle u, v \rangle_R = u^\top R v, \quad \|u\|_R = \sqrt{\langle u, u \rangle_R}, \quad (5)$$

$$\langle y, e \rangle_Q = y^\top Q e, \quad \|y\|_Q = \sqrt{\langle y, y \rangle_Q}, \quad (6)$$

where R and Q are weighting parameter matrices with appropriate dimensions. The constraint on the ILC controller is defined as

$$\Omega = \{u \mid -\bar{u}^c \leq u(t) \leq \bar{u}^c, t \in [0, N-1]\}, \quad (7)$$

where \bar{u}^c is the saturation constraint value designed for the ILC controller.

When the constraint on the ILC controller is stationary, the constrained ILC process can be transferred as alternating projections between the following two sets:

$$M_1 = \{(e, u) \in H \mid e = r - G u\}, \quad (8)$$

$$M_2 = \{(e, u) \in H \mid e = \mathbf{0}, u \in \Omega\}, \quad (9)$$

where M_1 is employed to represent the set of system dynamics and M_2 is the tracking goal under constraints. According to the definition of input and output space in (5) and (6), the Hilbert space H has the following form:

$$H = l_2[1, N] \times l_2[0, N-1], \quad (10)$$

where

$$\begin{aligned} \langle (e, u), (y, v) \rangle_H &= (e - y)^\top Q (e - y) \\ &\quad + (u - v)^\top R (u - v), \end{aligned} \quad (11)$$

$$\|(e, u)\|_H = \sqrt{\langle (e, u), (e, u) \rangle_H}. \quad (12)$$

Then, applying alternating projection between M_1 and M_2 , i.e., minimizing the distances between $(\mathbf{0}, u) \in M_2$ and $(e, u) \in M_1$, the constraint-aware ILC can be given as follows:

$$u_{k+1} = P_\Omega(f(P_\Omega(u_k), e_k)), \quad (13)$$

where $f(\cdot)$ is a specific unconstrained ILC update law and the projection operator $P_\Omega(\cdot)$ is defined as

$$P_\Omega(v) = \arg \min_{u \in \Omega} \|u - v\|_R^2. \quad (14)$$

When the constraint is active, the constraint-aware ILC design can avoid useless iterations during the ILC process. The convergence analysis of the constraint-aware ILC is equivalent to proving the convergence of alternating projection between M_1 and M_2 when they are not intersected. In this case, the following lemma demonstrates that alternating projections between two non-intersecting convex closed sets can converge to the minimum distance, which is denoted by $d(M_1, M_2)$.

Lemma 1 *M_1 and M_2 are closed convex sets in a Hilbert space. A point in the Hilbert space is denoted by $z = (e, u) \in H$. If $M_1 \cap M_2 \neq \emptyset$, the projection point sequence $\{z_k\}_{k \geq 0}$, defined by $z_{k+1} = P_{M_1/M_2}(z_k)$, converges in norm, i.e.,*

$$\lim_{k \rightarrow \infty} \|z_{k+1} - z_k\| = d(M_1, M_2). \quad (15)$$

Proof. Please refer to [22, 23] and the detailed proof is omitted here. ■

When the constraint on the ILC controller is user-decided in the closed-loop design, the constraint-aware value \bar{u}^c in the constraint set Ω should be determined during the constrained ILC design. In this case, the following two-variable optimization problem should be solved, i.e.

$$\begin{aligned} \min_{u_{k+1}, \bar{u}_{k+1}^c} & J_{k+1}(u_{k+1}, \bar{u}_{k+1}^c) \\ \text{s.t. } & e_{k+1} = S r - G P_{\Omega_{k+1}}(u_{k+1}), \end{aligned} \quad (16)$$

Algorithm 1 Constraint-aware ILC

Initialization: Given the system parameter matrices $\{A, B, C\}$, a reference trajectory r , an active actuator saturation \bar{u} , weighting parameter matrices Q and R , the initial feedforward ILC input $u_0 = \mathbf{0}$ and the initial constraint set Ω_0 where $\bar{u}_0^c = \bar{u}$. Set $k = 0$ and the maximum trial index k_{\max} .

Repeat:

- Calculate u_{k+1} by an unconstrained ILC update $f(\cdot)$.
- Set $u_{k+1} = P_{\Omega_k}(u_{k+1})$.
- For $t \in [0, N-1]$, set $u_{k+1}(t)$ as the variable \bar{u}_{k+1}^c to get a unknown vector $u_{k+1}[\bar{u}_{k+1}^c]$.
- Minimize the cost function $J_{k+1}(u_{k+1}[\bar{u}_{k+1}^c])$ to get \bar{u}_{k+1}^c .
- Set $u_{k+1} = P_{\Omega_{k+1}}(u_{k+1})$.
- Execute trial $k+1$ to obtain e_{k+1} .
- Set $k \rightarrow k+1$.

Until: $k = k_{\max}$.

Return: The feedforward signal $u_{k_{\max}}$ and the constraint-aware value \bar{u}^c for each trial.

where $J(\cdot)$ is a user-decided cost function. The constraint set is subject to the trial-varying \bar{u}_{k+1}^c , i.e., $\Omega_{k+1} = \{u | -\bar{u}_{k+1}^c \leq u(t) \leq \bar{u}_{k+1}^c, t \in [0, N-1]\}$ and \mathcal{S} is the impulse response matrix of *sensitivity* (from $r(t)$ to $e_k(t)$).

Algorithm 3 is given to solve this problem. Given an unconstrained ILC update law, Algorithm 3 can autonomously determine the constraint on the ILC controller in the closed-loop design with actuator saturation constraints.

In the next section, Algorithm 3 is verified on a numerical example to show its effectiveness.

4 Numerical Simulation

The effectiveness of the constraint-aware ILC is verified in a numerical simulation by employing NOILC as the unconstrained ILC update law. The effectiveness is demonstrated by the comparisons between applying NOILC in the closed-loop design without constraint awareness as in Fig. 2(a) and Algorithm 3 as in Fig. 2(b).

The plant model \mathcal{H} is given as

$$\mathcal{H}(s) = \frac{0.12s + 235}{0.00009s^4 + 0.01092s^3 + 21.385s^2}, \quad (17)$$

with its stabilizing feedback controller \mathcal{C} . The sampling time is 0.001s. The reference trajectory is a third-order profile with $N = 4051$.

According to the definition of the Hilbert space H , define the cost function of NOILC as

$$J_{k+1}(u_{k+1}) = \|e_{k+1}\|_Q^2 + \|u_{k+1} - u_k\|_R^2. \quad (18)$$

Then, the unconstrained NOILC is given as

$$u_{k+1} = u_k + Le_k, \quad (19)$$

where $L = (G^\top QG + R)^{-1} G^\top Q$.

In the simulation, the weighting matrices is chosen as $Q = qI$ and $R = rI$ where $q = 10^3$ and $r = 10^{-3}$. The initial constraint-aware value is chosen as the actuator saturation, i.e., $\bar{u}_0^c = \bar{u}$. The simulation conduct NOILC and Algorithm 3 for 30 trials, and hence $k_{\max} = 30$.

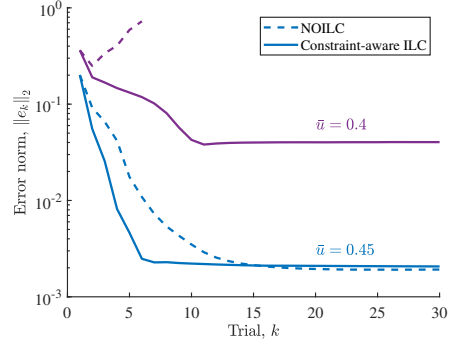


Fig. 3: Error convergence of the constraint-aware ILC under different actuator saturation constraints.

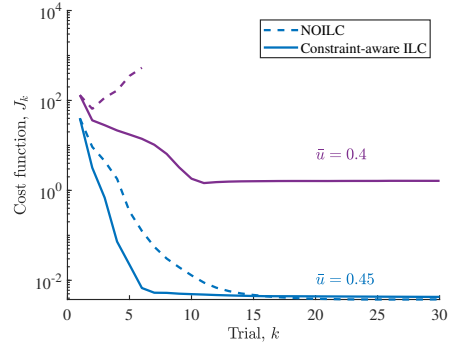


Fig. 4: Cost function of the constraint-aware ILC under different actuator saturation constraints.

The simulation results are given from Fig. 3 to Fig. 5. The error convergence of both NOILC and Algorithm 3 is shown in Fig. 3, where different actuator saturation values \bar{u} are tested. In both $\bar{u} = 0.45$ and $\bar{u} = 0.4$, the actuator constraints are all active. Algorithm 3 performs better than NOILC in the error convergence. In particular, NOILC is unstable in the trial domain when $\bar{u} = 0.4$ while Algorithm 3 still converges as k increases. In Fig. 4, the cost convergence of Algorithm 3 is also better than that of NOILC when $\bar{u} = 0.4$. In the first few trials, the cost has significant decrease while NOILC relatively reduces slower.

The variation of the constraint-aware value \bar{u}^c under different actuator saturation constraints is given in Fig. 5. The constraint-aware value tends to decrease in the first few tri-

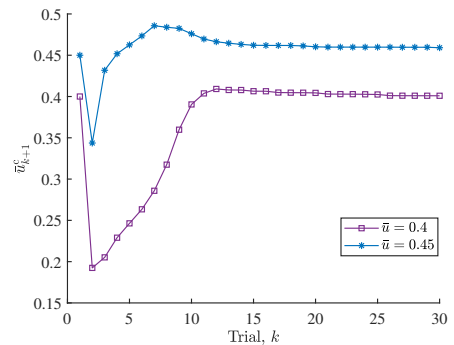


Fig. 5: Variation of the constraint-aware value \bar{u}^c by Algorithm 3 under different actuator constraints.

als and then converges to the given actuator saturation constraints.

5 Conclusion and Future Work

This paper solves an ILC input constraint problem in the closed-loop design with actuator saturation. The trial-domain integral windup would happen when without constraints on ILC. To solve this problem, a systematical consideration of the ILC input constraint problem in the closed-loop design is provided. A constraint-aware ILC design is given to autonomously determine the constraint on the ILC controller. Alternating projection framework is employed to give convergence analysis from a holistic perspective. Finally, compared to NOILC without constraint awareness, the effectiveness of the developed constraint-aware ILC design, i.e., Algorithm 3, is verified on a numerical simulation.

For future work, the robustness against model uncertainty and external trial-varying disturbances should be investigated for practical applications of the constraint-aware ILC.

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