Alternating Projection-Based Optimal ILC for Linear Systems with Non-Uniform Trial Lengths under Input Constraints

Zhihe Zhuang
Jiangnan University
China
z.h.z.zhuang@tue.nl

Max van Meer
TU/e
Netherlands
m.v.meer@tue.nl

Hongfeng Tao
Jiangnan University
China
taohongfeng@hotmail.com

Dianqing Zhou
TU/e
Netherlands
d.zhou1@tue.nl

Tom Oomen
TU/e
Netherlands
t.a.e.oomen@tue.nl

1 Introduction

Iterative learning control (ILC) improves tracking performance trial by trial. In practical applications of ILC, the repetitive process may end up early by accident and the actual trial length varies. The ultimate goal of dealing with non-uniform trial lengths in ILC is to try to make the tracking performance as good as the identical case. In other words, try to increase the convergence speed along the trial axis in view of less information for learning. In this paper, the optimal ILC framework is employed to solve this issue. The alternating (successive) projection method is modified to adapt to the trial-varying situation with input constraints.

2 Problem Formulation

The illustration of the non-uniform trial lengths in ILC is as follows. The actual trial length N_k is set to vary in $\{N_-, N_- + 1, \dots, N\}$, where N_- and N respectively denote the minimum and maximum lengths that occur in a particular application.

The lifted model with the same trial length N is employed, i.e.

$$y_k = Gu_k + d_k, \tag{1}$$

where G and d_k represent the system model and the effect of the initial conditions respectively. The discrete-time input and output sequence $u_k \in \mathbf{R}^{\ell N}$ and $y_k \in \mathbf{R}^{\mathrm{m}N}$. Define $y_d \in \mathbf{R}^{\mathrm{m}N}$ as the desired output. The tracking error vectors of systems with non-uniform trial lengths can be written as

$$e_k = F_k\left(y_d - y_k\right), \tag{2}$$

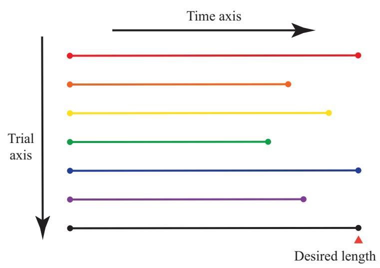

Figure 1: The illustration of the non-uniform trial lengths in ILC.

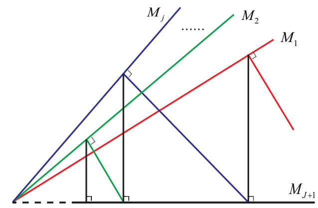

Figure 2: The illustration of designed alternating projections.

where

$$F_k = \begin{bmatrix} I_{N_k} \otimes I_{\mathrm{m}} & 0 \\ 0 & 0 \otimes I_{\mathrm{m}} \end{bmatrix}, \tag{3}$$

and I_l denotes the identity matrix with dimensions $l \times l$, and $\otimes$ denotes the Kronecker product.

3 Alternating Projection Design

Different from [1], multiple sets are introduced to represent the actual dynamics of the varying trial lengths. The ILC problem is equivalent to iteratively finding a point in the intersection of the following multiple sets in Hilbert space H

$$M_j = \left\{(e,u) \in H : e = F_j(y_d - y), y = Gu + d\right\}, \tag{4}$$
$$M_{J+1} = \left\{(e,u) \in H : e = 0, u \in \Omega\right\}, \tag{5}$$

where M_j and M_{J+1} respectively represent system dynamics and the tacking objective. Ω is the input constraint set. Then, the projection order is designed as (6) and the illustration can be seen in Fig. 2.

$$M_{j_k} = \begin{cases} M_j \in \{M_1, M_2, \dots, M_J\}, & k \text{ is odd}, \\ M_{J+1}, & k \text{ is even}. \end{cases} \tag{6}$$

4 Ongoing Research

Future work includes the implementation of projections and developing optimal ILC algorithms for systems with non-identical trial lengths under input constraints.

References

[1] Y. Chen, B. Chu, and C.T. Freeman, "Generalized Iterative Learning Control Using Successive Projection: Algorithm, Convergence, and Experimental Verification," *IEEE Trans. Control Syst. Technol.*, vol. 28, no. 6, pp. 2079-2091, 2020.